Influence of the polymeric interlayer shear modulus in the laminated glass panels transmission loss

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Abstract
The need for precise predictions of acoustic isolation is increasingly growing due to the application of current regulations, focused on guaranteeing a certain comfort (or health) level for final users. This implies serious difficulties, mainly because experimental data obtained from laboratory measurements (standardised transmission chamber) deviate from on site measurements, due to contributions from indirect propagation and particular boundary conditions. The relatively large uncertainty of on site measurements has to be taken into account as well.

In addition to this, European (CEN) standards for glass in building do not consider any procedure for acoustic isolation predictions on multilayer glazing (laminated glass with polymeric films) and multiple glazing (several multilayer glazings separated by intermediate gas gaps). Only ISO/PAS 16940 standard suggests a calculation model for laminated glass, which results clearly insufficient. The present work presents more elaborated procedures for acoustic isolation simulation by means of analytic models which compare successfully with experimental results and avoid the use of other techniques which require a higher cost both in human and technical resources, such as the finite elements method.

1. Introduction
The uncertainty in the determination of transmission loss (TL) of sandwich panel-like structures -laminated glass- is partly due to the uncertainties on the vibrational and mechanical properties of their components coming from the fact that, in particular, polymeric films do not co-exist with identical properties independently from the multilayer structure. Essentially, the parameters defining acoustic behaviour are bending stiffness and loss factor of the whole system. This work studies the influence of the mechanical properties of the elements in the multilayer on these parameters and on the acoustic isolation, by means of a constitutive model of two elastic layers and a viscoelastic nucleus which permits the shear deformation, through a general RKU (Ross-Kerwin-Unger, 1959 [6]) analysis. This study will also be performed by means of another method, based on impedance coupling (Ookura-Saito, 1978 [2]), from which, although the influence of the terms related to layer coupling on the global stiffness is not considered, the magnitude of these complementary terms can be determined from a non linear fit to experimental data, under the assumption of a non-frequency dependent global parameters model.

2. Prediction models for acoustic isolation
Although other models have been applied [4,5,10], we will simplify by focusing on the afore-mentioned Ookura-Saito model [2], which is detailed in what follows for the case of a non permeable multi-layered panel.

Within this model, the sound transmission through a multiple wall structure (figure 1) for an incidence surface, \( p_{\text{N2}} \), and incidence angle \( \theta \) is studied by analysing the relation between pressures at each incidence surface, \( p_{\text{N2}} \), and incidence pressure \( p_{\text{N1}} \). For the analysis, each physical parameter of the i-th element is numbered with the sub-index i, while a second sub-index is used in order to indicate the side of the element to be considered (1-right, 2-left).

Being \( Z_{i2} \) the normal acoustic impedance from the left side of the Nth element surface and \( Z_{i1} \) the free field normal acoustic impedance of a surface with oblique incidence, which is equal to the impedances relation of the first element, \( Z_{12} \), (\( \rho \) being the air density, in Kg/m\(^2\), and \( c \) is the speed of sound in air, in m/s),

\[
P_{\text{N2}} = \frac{p_{\text{N1}} + p_{\text{r1}}}{p_{\text{r1}}} = \frac{2Z_{\text{N2}}}{Z_{\text{N2}} + \frac{\rho c}{\cos \theta}} \tag{1}
\]

Taking into account the pressure conditions at each surface, the expression for the transmission factor at normal incidence is given by:

\[
\tau(\theta) = \frac{p_{\text{r1}}}{p_{\text{r2}}} = \frac{p_{\text{r2}}}{p_{\text{r1}}} = \frac{p_{\text{r2}}}{p_{\text{r1}}} = \frac{p_{\text{r1}}}{p_{\text{r2}}} = \ldots = \frac{p_{\text{r2}}}{p_{\text{r1}}} \tag{2}
\]

And the diffuse field transmission, for a limit angle \( \theta_{\text{lim}} \) will be given by:

\[
\tau_d = \frac{1}{\theta_{\text{lim}}} \int_0^{\theta_{\text{lim}}} \tau(\theta) \cos \theta \sin \theta d\theta \tag{3}
\]

For a simple wall or non permeable layer, it is considered that vibrations at an infinite plate of width \( h \) induce a pressure difference at both sides of the plate. In this case, the velocity of the plate in the transversal direction can be written as:
where \( p_i \) is the incident sound pressure on the surface at \( x=0 \), \( p_t \) is the transmitted pressure at \( x=h \) and \( Z_m \) is the so-called surface impedance of the plate, which is given, to a first approximation, by,

\[
Z_m = 2\pi\eta m \left( \frac{f}{f_c} \right)^2 \sin^2\theta + j2\pi\eta m \left( 1 - \left( \frac{f}{f_c} \right)^2 \sin^2\theta \right)
\]  

where \( \eta \) is the loss factor (dimensional), \( f_c \) is the critical frequency of the panel (in Hz), and \( m \) is the mass per surface unit (in Kg/m²).

Several partitions formed by several layers may be found within the problem we are facing:

A) Monolithic glass, with standardised characteristics, which acoustically behaves as a permeable layer.

B) Polymeric films, usually PVB (PolyVinyl Butiral), with organic components, or PMMA (PolyMethyl MetAcralate), which derive from metacrilate resins. From an acoustic point of view, both of them attenuate vibration transmission (damping).

It should be noted that glass thickness is significantly larger than that of intermediate polymeric films, and that a large number of configurations include a three-layer structure with two glass panels of the same thickness (symmetric laminate).

The standard ISO/PAS 16940:2004 provides a method for the calculation of transmission losses, \( TL = 10\log(1/\tau(\theta)) \) in this kind of partitions. In particular:

\[
\tau(\theta) = \frac{\tau_{\text{Trans}}}{\tau_{\text{Inc}}} = \frac{p_t^2}{p_i^2} = 
\left[ 1 + \eta \left( \frac{\omega B}{2pc} \cos\theta \left( \frac{\omega^2 B}{c^3} \sin^2\theta \right) \right)^2 + \left( \frac{\omega B}{2pc} \cos\theta \left( 1 - \frac{\omega^2 B}{c^3} \sin^2\theta \right) \right)^2 \right]^{-1}
\]

Where \( I \) is the sound intensity (W/m²), \( \rho_s \) the surface density of the plate (kg/m²), and the rest of the parameters have been previously defined. The standard also includes an experimental procedure which permits to obtain the equivalent stiffness modulus of a laminated glass, dependent on the frequency \( B=B(f) \) and the loss factor. In order to obtain the diffuse field transmission coefficient, the standard proposes a limit angle of 75°.

However, this standardised expression has less utility (and a lower quality) as predictive model than the models based on impedance coupling, as the one previously exposed, due to the fact that it does not consider the independent contribution from each layer, while it defines the structure from global equivalents for bending stiffness and loss factor. These equivalents are strictly obtained experimentally, which limits their application in practice, during the design phase of the glazing system. This is why the problem of improving the predictive capabilities of the model is treated here, by including a constitutive model for the material that permits generating isolation curves from the vibrational and mechanical parameters (\( B \) and \( \eta \)) of the constituent layers.

To a first approximation one can introduce an equivalent stiffness bending factor and an equivalent loss factor in which there is no influence from the coupling between layers (as it is the case of Ookura-Saito model for three layers), from the following expressions:

\[
B_{eq} = \sum_i B_i, \quad \eta_{eq} = \frac{\sum_i \eta_i m i}{\sum_i m i^2}
\]

In the following graph (figure 2), the prediction for isolation from both models is shown, for the case of a laminated glass “44.2 acoustic PVB” (two 4mm nominal thickness glass panels joined by a 0.76 mm PVB film specially designed for acoustic isolation). Experimental results obtained by means of a transmission chamber are also shown for comparison.

Loss factors of 1.4 and 0.1 have been taken for PVB and glass, respectively. The improvement in the predictions from the Ookura Saito three-layer model -O&S (3c) in the legend- is clear in comparison with the non permeable layer model from the standard -ISO/PAS-, even though both models neglect, a priori, the contribution of layer couplings to the stiffness and damping of the whole system. The differences between theoretical and experimental -Exp.- results can be attributed to boundary conditions (frames and fixing conditions) at the low frequencies. For symmetric glass panes (as the one in the example) the application of the one layer Ookura-Saito model leads to a similar result, due to the coincidence of the critical frequency of both glass panels.

3. Influence of layer coupling on acoustic isolation.

It is known that the main advantage of incorporating intermediate layers (polymeric films) with viscoelastic behaviour is that very high loss factors can be reached even with thin layers, in addition to the fact that they do not appreciably increase the stiffness of the whole system. This effect implies that bending wavelength is relatively short, compared to that of adhered layers, such that damping increases relatively fast with thickness.

When a sandwich-like structure is put through a bending cycle, bending deformations are produced on the
plate. The exterior layers, more rigid, induce compression strengths on the intermediate layer, which gives place to shear stresses in the longitudinal plane. This mechanism is responsible for the vibrational energy dissipation.

In order to operate correctly, the material used for ensuring damping should be neither too rigid, which would produce an almost perfect coupling, nor too soft so that it would deform excessively, which would also lead to a too high coupling.

This idea will be implemented in our simulation model by using a constitutive model for the material which will analyse the explicit dependence on shear deformation through shear modulus. Both the loss factor and the bending stiffness of the whole system will be taken as frequency dependent, showing a closer behaviour to experimental results in the high frequency range (at low frequencies only bending waves show an effective propagation, not present for shear waves, so that plate behaviour is described by means of constant values for stiffness and damping, as can be checked from three-layer model shown in figure 2).

This problem will be firstly treated from a generalization of the three-layer Ookura-Saito model, which does not allow for a direct modelling of the contribution of coupling terms between layers, but nevertheless it permits the quantification of these terms, under the simplifying hypothesis of the model. If we consider the impedance of a non permeable three layer configuration

$$Z_m = \sum_{i=1}^{3} Z_{m_i} = \sum_{i} \eta_i 2\pi f_m \left( \frac{f}{f_i} \right)^2 \sin^4 \theta + j2\pi f_m \left( 1 - \frac{f}{f_i} \right)^2 \sin^4 \theta$$

Which is a complex magnitude, $Z_m=Z_{mR}+jZ_{mi}$, whose real part depends directly on the loss factor, while imaginary part is a contribution due to mass and flexural angle. Due to the particularities of usual polymeric films (PVB), for which $f_c = 10^6$ Hz, its direct influence is numerically negligible, as, in the most favourable case ($f=5000$ Hz), $\eta_{m_i}(f/f_c)^2=10^4$ Kg/m$^2$, so that the term related to the critical frequency of the film makes the contribution from bending not relevant and only the contribution from mass remains.

In order to generalise the model by including contributions to the system's stiffness due to layer coupling, taking into account the relation for elastic plates between stiffness and critical frequency, the impedance components can be re-written as follows (using the sub-index 'eq' for total magnitudes):

$$R_m = 2\pi f^2 \sin^4 \theta \left( \frac{2\pi}{c^2} \right)^2 \sum_i \eta_i B_i$$

$$X_m = 2\pi f^2 m_0 - 2\pi f^2 \sin^4 \theta \left( \frac{2\pi}{c^2} \right)^2 B_{eq}$$

In order to evaluate numerically the influence of the remaining contributions to the stiffness of the system, that is to say, the resistive losses which represent the role of elastic elements of layer coupling, we will introduce a grouping term for these contributions, $R_{eq}(f)$. Its magnitude will be evaluated by means of a fit minimizing the quadratic error between the predictive model and the experimental measurements for transmission losses. Therefore, we introduce it in the impedance for the simplified model,

$$R_m = 2\pi f^2 \sin^4 \theta \left( \frac{2\pi}{f_i} \right)^2 m_1 + \eta_m \left( \frac{f}{f_i} \right)^2 + R_{eq}(f)$$

The constants have been introduced by convenience. The sub-index ‘ac’ makes reference to the coupling parameters between plates, i.e., the “missing” loss factor and bending stiffness.

This estimation will be performed by means of an inversion method, from transmission chamber measurements, with the minimisation of the following quadratic error function:

$$R_{eq}(f) = \frac{(2\pi)^2}{c^2} \eta_{ac} B_{eq} f^2 = Af^2$$
where \( \tau_{\text{d},i} \) is the transmission coefficient measured at i-th frequency and \( \tau_{\text{d},i}^* \) is the estimation coming from the equation for the diffuse field transmission coefficient. The optimisation process leads to the following equation system in partial derivatives for the variables which are to be minimised. In our case (10), we propose searching for \( \eta_i, \tau_{\text{c}} \) (or equivalently B) parameters for glass, and for A factor for layer coupling. The same loss factor is taken for glass panes.

In this way, the equation system can be written in the form:

\[
\frac{\partial e}{\partial \eta} = 2 \sum_{i=1}^{N} (\tau_{\text{d},i} - \hat{\tau}_{\text{d},i}) \frac{\partial \hat{\tau}_{\text{d},i}}{\partial \eta} = 0 \\
\frac{\partial e}{\partial \eta} = 2 \sum_{i=1}^{N} (\tau_{\text{d},i} - \hat{\tau}_{\text{d},i}) \frac{\partial \hat{\tau}_{\text{d},i}}{\partial \eta} = 0 \\
\frac{\partial e}{\partial A} = 2 \sum_{i=1}^{N} (\tau_{\text{d},i} - \hat{\tau}_{\text{d},i}) \frac{\partial \hat{\tau}_{\text{d},i}}{\partial A} = 0 \\
\frac{\partial e}{\partial f_{\text{c},3}} = 2 \sum_{i=1}^{N} (\tau_{\text{d},i} - \hat{\tau}_{\text{d},i}) \frac{\partial \hat{\tau}_{\text{d},i}}{\partial f_{\text{c},3}} = 0
\]

Equations (11) and (12) form a non linear equation system whose resolution provides with the value of the variables minimising the error involved. The complexity of the system suggests the resolution of the non linear equation systems [8-9] involved by applying an iteration method. The iteration models are described in the corresponding references. The best results have been obtained by using Broyden’s method and Newton’s method (however, this is more computationally expensive due to the Jacobian determinant inversion). The experimental isolation measurements have been taken from different laboratories, so the homogeneity of materials (and, therefore, their vibromechanic characterisation) and measurements can not be guaranteed. However, this fact does not seem relevant given the obtained results and the purpose of the calculation, which is the evaluation of the magnitude of the \( R(f) \) contribution. The following graphs (figure 3) show the obtained fits, together with the experimental measurements, and the values for the variables for some of the configurations studied.

For all the studied cases, \( A=10^{-9} \) \( \text{sKg/m}^2 \), so one can consider this contribution as numerically negligible and conclude that the modified model (10) does not give significant improvements with respect to the three-layer Ookura-Saito. We can then conclude that layer coupling terms are not relevant for predictive isolation models where vibromechanical parameters for the layers, B, and \( \eta \) are not frequency dependent. Another possibility could consist in including in the simulation, as stated at the beginning of this section, a constitutive model for the material involving a suitable frequency dependence of the vibromechanical parameters, based on an approximation to the real elastic behaviour, as shear deformation would then be allowed. One could expect in this case, as previously indicated, an improvement in the prediction for high frequencies. An introduction to this problem is exposed within the following section.

### 44.2 PVB Acústico / Acoustic

| \( \eta_1 \) | 0.2632 |
| \( B_1 \) | 1.1691 \( \times 10^{-7} \) \( \text{Nm} \) |
| \( f_{\text{c},1} \) | 1.7318 \( \times 10^{-3} \) \( \text{Hz} \) |
| \( A \) | 1.7562 \( \times 10^{-9} \) \( \text{sKg/m}^2 \) |

### 54.2 PVB Estándar / Standar

| \( \eta_1 \) | 0.1836 |
| \( B_1 \) | 2.2465 \( \times 10^{-7} \) \( \text{Nm} \) |
| \( f_{\text{c},1} \) | 1.2493 \( \times 10^{-3} \) \( \text{Hz} \) |
| \( B_3 \) | 2.1135 \( \times 10^{-3} \) \( \text{Nm} \) |
| \( f_{\text{c},3} \) | 1.5616 \( \times 10^{-7} \) \( \text{Hz} \) |
| \( A \) | 3.1975 \( \times 10^{-9} \) \( \text{sKg/m}^2 \) |

### 54.2 PVB Acústico / Acoustic

| \( \eta_1 \) | 0.3010 |
| \( B_1 \) | 1.7900 \( \times 10^{-7} \) \( \text{Nm} \) |
| \( f_{\text{c},1} \) | 1.3995 \( \times 10^{-3} \) \( \text{Hz} \) |
| \( B_3 \) | 1.4320 \( \times 10^{-3} \) \( \text{Nm} \) |
| \( f_{\text{c},3} \) | 1.7494 \( \times 10^{-7} \) \( \text{Hz} \) |
| \( A \) | 1.6177 \( \times 10^{-9} \) \( \text{sKg/m}^2 \) |

Figure 3
Fits obtained by inversion method
4. Constitutive models for the material.

The first analytic approximation to this problem is found in Cremer-Heckl (1973) [3], where for the case of a three-layer laminated structure, with viscoelastic nucleus of smaller width, it is shown that,

\[
\eta_{eq} = \eta_2 + \frac{2E_2h_2a^2g}{B(1 + jn_2)q}.
\]

\[
B_{eq} = 2B \left[ 1 + \frac{E_2h_2a^2g}{B(1 + jn_2)q} \right] = 2B \left( 1 + \frac{3g}{1 + g} \right)
\]

where \( E_2 \) is the Young modulus for the glass (N/m²) and \( h_2 \) is the thickness of the i-th layer (m). We have taken \( B_i = E_i d_i / 12 \) and \( a = d_1 / 2 \). The shear parameter, \( g \), explicitly depends on the shear modulus of the intermediate film, \( G_2 \), through the expression,

\[
g = \frac{2G_2}{E_2h_2h_k^2}, \quad \text{where} \quad k = \sqrt{\omega m_{eq} / B_{eq}}
\]

which governs the vibrational energy dissipation phenomenon, indicated at the beginning of previous section.

Wave number, \( k \), corresponds to the propagation of a bending wave in an elastic plate, with angular frequency \( \omega \), following a differential equation of the form,

\[
B_{eq} \frac{\partial^2 z}{\partial x^2} + m_{eq} \frac{\partial^2 z}{\partial t^2} = f(x,t)
\]

where \( z, x, \) and \( t \) are the displacement of the plate, the propagation direction and time, respectively.

However, a more detailed studied can be found in Ross-Kerwin-Ungar (1959) [6] for the construction of a homogeneous bending stiffness for the three layer system, \( B_{eq} \). This model is based on similar hypothesis for the bending wave as in the previous model, but with a more refined geometric construction. Within this model we can write,

\[
B_{eq} = K_1 h_{12}^2 + K_2 h_{21}^2 + K_3 h_{32}^2 - K_2 \left( \frac{h_{21}^2}{12} \frac{\partial^2 \psi}{\partial \phi^2} \right) + K_1 D^2 + K_2 (h_{21} - D)^2 + K_3 (h_{31} - D)^2
\]

\[
+ K_3 (h_{31} - D)^2 - h_2 \left( \frac{\partial \psi}{\partial \phi} \right) \left( \frac{K_2}{2} (h_{21} - D) + K_3 (h_{31} - D) \right)
\]

where \( K_i = E_i h_i \) is the thickness of the \( i \)-th layer, \( E_i \) its Young modulus, \( h_i \) the distance between neutral planes of layers \( i, j \), \( D \) is the distance between the neutral plane of layer 1 and the symmetry axis of the same layer, and \( d / d \) is the relation between shear angle and bending angle of layer 2.

Within the model, by geometric construction, a relation between the distance to the neutral axe and the bending angle can be established [6]. Shear parameter, \( g \), depends explicitly on the shear modulus of the intermediate film, \( G_2 \), through the following equation (which is by derivation very similar to the previous model)

\[
g = \frac{G_2}{E_2h_2h_k^2}
\]

The wave number is defined under the same conditions for bending wave propagation. A study of the direct influence of shear modulus of the film by means of this model can be found in [7].

It is evident that \( B_{eq} = B_{eq}(\omega) \) as a consequence of \( g \) parameter. The first three terms coincide with (6) and (9), that is, with the value for stiffness of the whole system used in models where no coupling between layers is considered. The remaining terms in equation (18) model, under the geometric hypothesis used for building the model, the contribution of the layer coupling to the stiffness of the whole system.

The so-derived homogenised stiffness modulus for the system can be incorporated to the predictive models equations, in order to determine transmission losses with a better matching to real behaviour. However, the following limitations to the applicability of the model must be taken into account:

A) The influence of elastic parameters of the system and their relation with the frequency value given by (17) must be appreciable within the high frequency interval and around the drop in isolation values at the critical frequency of the panel, as for lower frequencies there is no shear wave propagation.

B) RKU (and also Cremer-Heckl) is a prismatic piece model, applicable to cylindrical deformations of plates,
following (15), and which induces the wave number, together with the relation with the frequency of the model. However, the experimental measurements are performed by means of a standardised transmission chamber. From a vibromechanical point of view, the previous model is not applicable for a thin plate under four edges fixing boundary conditions, which, under particular conditions, shows a non-linear behaviour due to membrane strains.

About the implementation of the model, we can outline the fact that $\beta_{in} = \beta_{in}(k)$ and $k = k(\beta_{in})$ form a non linear system of coupled equations which must be solved by iteration.

Figure 5 shows the graphs for the predictive Ookura-Saito model for three uncoupled layers -O&S(3c) in the legend-, and Ookura-Saito for one layer with coupling terms obtained from equations (13) for $\eta_1$ and (16) for $\beta_{in}$ -O&S(1c)+Ac-. There is a remarkable qualitative improvement in the predictions of this last model, with a closer behaviour to the experimental data, as the compensation of drop isolation around coincidence frequencies and sensitivity to the vibromechanical parameters variations.

5. Conclusions

As has been analysed in sections 2 and 3, it is possible to predict acoustic isolation values for laminated glass from simulation models based on impedance coupling, in good agreement with experimental results. The exposed models give considerably better results than the model proposed by the corresponding standard. However, this kind of models is not sensitive enough to predict all the phenomenology involved in isolation processes for this kind of structures, specially in the case of damping around the critical frequencies of the panels, due to the presence of a viscoelastic film appropriately chosen for this function. This improvement can be achieved by using a constitutive theory for the multilayer structure which models, as seen in section 4, layer coupling and shear stress of the structure during vibration processes, these phenomena participating actively in energy dissipation.

The numerical simulation shows the expected qualitatively better prediction, with a more appropriate sensitivity to variations of the vibromechanical parameters of the constituents. As an example, a table with the estimation of the global index $R_w$ (following EN ISO 717) for the same laminated glass types of figure 5 has been included in figure 6. The model including coupling terms predicts a 1 dB variation between the two types of glass panes (acoustic PVB giving better results, in front the 2 dB of the experiment), and it’s more accurate. Oppositely, the uncoupled layer model is not sensitive to the variation of global parameters. Other constitutive models consistent with the experimental (or boundary) measurement conditions to be described (in this case, conditions at a standardised transmission chamber) must be developed in order to achieve a quantitative improvement in the a priori isolation estimations. This involves serious difficulties due to the non-existence of analytic solutions for the theoretical treatment of plates under non-linear conditions. Additionally, predicted results can be improved with a more precise vibromechanical characterisation of the materials involved (design values have been used for the simulation).

6. References